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# ASYMPTOTIC ESTIMATE FOR PERTURBED SCALAR CURVATURE EQUATION.

SAMY SKANDER BAHOURA

ABSTRACT. We consider the equation  $\Delta u_\epsilon = V_\epsilon u_\epsilon^{(n+2)/(n-2)} + \epsilon W_\epsilon u_\epsilon^\alpha$  with  $\alpha \in ]\frac{n}{n-2}, \frac{n+2}{n-2}[$  and we give some minimal conditions on  $\nabla V$  and  $\nabla W$  to have an uniform estimate for their solutions when  $\epsilon \rightarrow 0$ .

## 1. INTRODUCTION AND RESULTS.

We denote  $\Delta = -\sum_i \partial_{ii}$  the geometric Laplacian on  $\mathbb{R}^n, n \geq 3$ .

Let us consider on open set  $\Omega$  of  $\mathbb{R}^n, n \geq 3$ , the following equation:

$$\Delta u_\epsilon = V_\epsilon u_\epsilon^{(n+2)/(n-2)} + \epsilon W_\epsilon u_\epsilon^\alpha \quad (E_\epsilon)$$

where  $V_\epsilon$  and  $W_\epsilon$  are two regular functions and  $\alpha \in ]\frac{n}{n-2}, \frac{n+2}{n-2}[$ .

We assume:

$$0 < a \leq V_\epsilon(x) \leq b, \quad \|\nabla V_\epsilon\|_{L^\infty} \leq A \quad (C_1)$$

$$0 < c \leq W_\epsilon(x) \leq d, \quad \|\nabla W_\epsilon\|_{L^\infty} \leq B \quad (C_2)$$

**Problem:** Can we have an  $\sup \times \inf$  estimate with the minimal conditions  $(C_1)$  and  $(C_2)$  ?

Note that for  $W \equiv 0$ , the equation  $(E_\epsilon)$  is the wellknown scalar curvature equation on open set of  $\mathbb{R}^n, n \geq 3$ . In this case, there is many results about this equation, see for example [B] and [C-L 1].

When  $\Omega = \mathbb{S}_n$  YY. Li, give a flatness condition to have the boundedness of the energy and the existence of the simple blow-up points, see [L1] and [L2].

In [C-L 2], Chen and Lin gave a conterexample of solutions of the scalar curvature equation with unbounded energy. The conditions of Li are minimal in heigh dimension.

Note that, in [C-L 1] and [C-L 3], there is some results concerning Harnack inequalities of type  $\sup \times \inf$  with the "Li-flatness" conditions for the following equation:

$$\Delta u = V u^{(n+2)/(n-2)} + g(u)$$

where  $g$  is a regular function ( at least  $C^1$  ) such that  $g(t)/[t^{(n+2)/(n-2)}]$  is deacrising and tends to 0 when  $t \rightarrow +\infty$ . They extend Li result ([L1]) to any open set of the euclidian space.

We can find in [A], some existence results for the presribed scalar curvature equation.

In our work we have no assumption on the energy. We use the blow-up analysis and the moving-plane method, developped by Gidas-Ni-Nirenberg, see [ G-N-N]. This method was used by different authors to have a priori estimates, look for example, [B], [B-L-S] ( in dimension 2), [C-L 1], [C-L 3], [L 1] and [L 2].

We set  $\delta = [(n+2) - \alpha(n-2)]/2, \delta \in ]0, 1[$ . We have:

**Theorem 1.** For all  $a, b, c, d, A, B > 0$ , for all  $\alpha \in ]\frac{n}{n-2}, \frac{n+2}{n-2}[$  and all compact set  $K$  of  $\Omega$ , there is a positive constant  $c = c(a, b, c, d, A, B, \alpha, K, \Omega, n)$  such that:

$$\epsilon^{(n-2)/2(1-\delta)} (\sup_K u_\epsilon)^{1/3} \times \inf_\Omega u_\epsilon \leq c$$

for all  $u_\epsilon$  solution of  $(E_\epsilon)$  with  $V_\epsilon$  and  $W_\epsilon$  satisfying the conditions  $(C_1)$  and  $(C_2)$ .

Now, we suppose that  $V_\epsilon$  satisfies:

$$0 < a \leq V_\epsilon(x) \leq b \text{ and } \|\nabla V_\epsilon\|_{L^\infty(\Omega)} \leq k\epsilon \quad (C_3)$$

We have:

**Theorem 2.** For all  $a, b, c, d, k, B > 0$ , for all  $\alpha \in ]\frac{n}{n-2}, \frac{n+2}{n-2}[$  and all compact set  $K$  of  $\Omega$ , there is a positive constant  $c = c(a, b, c, d, k, B, \alpha, K, \Omega, n)$  such that:

$$\sup_K u_\epsilon \times \inf_\Omega u_\epsilon \leq c$$

for all  $u_\epsilon$  solution of  $(E_\epsilon)$  with  $V_\epsilon$  and  $W_\epsilon$  satisfying the conditions  $(C_3)$  and  $(C_2)$ .

Note that in [B], we have some results as the previous but for prescribed scalar curvature equation with subcritical exponent tending to the critical. Here, we have a  $\sup \times \inf$  inequality for the scalar curvature equation, with critical exponent, perturbed by a nonlinear term. We can see the influence of this non-linear term.

## 2. PROOFS OF THE THEOREMS.

### Proof of the theorem 1.

Without loss of generality, we suppose  $\Omega = B_1$  the unit ball of  $\mathbb{R}^n$ . We want to prove an a priori estimate around 0. We can also suppose  $\epsilon \rightarrow 0$ , the case  $\epsilon \not\rightarrow 0$  is solved in [B].

Let  $(u_i)$  and  $(V_i)$  be a sequences of functions on  $\Omega$  such that:

$$\Delta u_i = V_i u_i^{(n+2)/(n-2)} + \epsilon_i W_i u_i^\alpha, \quad u_i > 0,$$

with  $0 < a \leq V_i(x) \leq b, 0 < a \leq W_i(x) \leq d, \|V_i\|_{L^\infty} \leq A$  and  $\|W_i\|_{L^\infty} \leq B$ .

We argue by contradiction and we suppose that the  $\sup \times \inf$  is not bounded.

We have:

$\forall c, R > 0 \exists u_{c,R}$  solution of  $(E_1)$  such that:

$$\epsilon^{(n-2)/2(1-\delta)} R^{n-2} \left( \sup_{B(0,R)} u_{\epsilon,c,R} \right)^{1/3} \times \inf_\Omega u_{\epsilon,c,R} \geq c, \quad (H)$$

### Proposition : (blow-up analysis)

There is a sequence of points  $(y_i)_i, y_i \rightarrow 0$  and two sequences of positive real numbers  $(l_i)_i, (L_i)_i, l_i \rightarrow 0, L_i \rightarrow +\infty$ , such that if we set  $v_i(y) = \frac{u_i(y + y_i)}{u_i(y_i)}$ , we have:

$$0 < v_i(y) \leq \beta_i \leq 2^{(n-2)/2}, \quad \beta_i \rightarrow 1.$$

$$v_i(y) \rightarrow \left( \frac{1}{1 + |y|^2} \right)^{(n-2)/2}, \text{ uniformly on all compact set of } \mathbb{R}^n.$$

$$l_i^{(n-2)/2} \epsilon_i^{(n-2)/2(1-\delta)} [u_i(y_i)]^{1/3} \times \inf_{B_1} u_i \rightarrow +\infty,$$

### Proof of the proposition:

We use the hypothesis  $(H)$ , we take two sequences  $R_i > 0, R_i \rightarrow 0$  and  $c_i \rightarrow +\infty$ , such that,

$$\epsilon_i^{(n-2)/2(1-\delta)} R_i^{(n-2)} \left( \sup_{B(0, R_i)} u_i \right)^{1/3} \times \inf_{B_1} u_i \geq c_i \rightarrow +\infty,$$

Let  $x_i \in B(x_0, R_i)$  be a point such that  $\sup_{B(0, R_i)} u_i = u_i(x_i)$  and  $s_i(x) = (R_i - |x - x_i|)^{(n-2)/2} u_i(x)$ ,  $x \in B(x_i, R_i)$ . Then,  $x_i \rightarrow 0$ .

We have:

$$\max_{B(x_i, R_i)} s_i(x) = s_i(y_i) \geq s_i(x_i) = R_i^{(n-2)/2} u_i(x_i) \geq \sqrt{c_i} \rightarrow +\infty.$$

We set:

$$l_i = R_i - |y_i - x_i|, \quad \bar{u}_i(y) = u_i(y_i + y), \quad v_i(z) = \frac{u_i[y_i + (z/[u_i(y_i)]^{2/(n-2)})]}{u_i(y_i)}.$$

Clearly we have,  $y_i \rightarrow x_0$ . We also obtain:

$$L_i = \frac{l_i}{(c_i)^{1/2(n-2)}} [u_i(y_i)]^{2/(n-2)} = \frac{[s_i(y_i)]^{2/(n-2)}}{c_i^{1/2(n-2)}} \geq \frac{c_i^{1/(n-2)}}{c_i^{1/2(n-2)}} = c_i^{1/2(n-2)} \rightarrow +\infty.$$

If  $|z| \leq L_i$ , then  $y = [y_i + z/[u_i(y_i)]^{2/(n-2)}] \in B(y_i, \delta_i l_i)$  with  $\delta_i = \frac{1}{(c_i)^{1/2(n-2)}}$  and  $|y - y_i| < R_i - |y_i - x_i|$ , thus,  $|y - x_i| < R_i$  and,  $s_i(y) \leq s_i(y_i)$ . We can write:

$$u_i(y)(R_i - |y - y_i|)^{(n-2)/2} \leq u_i(y_i)(l_i)^{(n-2)/2}.$$

But,  $|y - y_i| \leq \delta_i l_i$ ,  $R_i > l_i$  and  $R_i - |y - y_i| \geq R_i - \delta_i l_i > l_i - \delta_i l_i = l_i(1 - \delta_i)$ . We obtain,

$$0 < v_i(z) = \frac{u_i(y)}{u_i(y_i)} \leq \left[ \frac{l_i}{l_i(1 - \delta_i)} \right]^{(n-2)/2} \leq 2^{(n-2)/2}.$$

We set,  $\beta_i = \left( \frac{1}{1 - \delta_i} \right)^{(n-2)/2}$ , clearly, we have,  $\beta_i \rightarrow 1$ .

The function  $v_i$  satisfies:

$$\Delta v_i = \tilde{V}_i v_i^{(n+2)/(n-2)} + \epsilon_i \tilde{W}_i \frac{v_i^{n/(n-2)}}{[u_i(y_i)]^{[(n+2)/(n-2)] - \alpha}}$$

where,  $\tilde{V}_i(y) = V_i [y + y/[u_i(y_i)]^{2/(n-2)}]$  and  $\tilde{W}_i(y) = W_i [y + y/[u_i(y_i)]^{2/(n-2)}]$ .

Without loss of generality, we can suppose that  $\tilde{V}_i \rightarrow V(0) = n(n-2)$ .

We use the elliptic estimates, Ascoli and Ladyzenskaya theorems to have the uniform convergence of  $(v_i)$  to  $v$  on compact set of  $\mathbb{R}^n$ . The function  $v$  satisfies:

$$\Delta v = n(n-2)v^{N-1}, \quad v(0) = 1, \quad 0 \leq v \leq 1 \leq 2^{(n-2)/2},$$

By the maximum principle, we have  $v > 0$  on  $\mathbb{R}^n$ . If we use Caffarelli-Gidas-Spruck result, (see [C-G-S]), we obtain,  $v(y) = \left( \frac{1}{1 + |y|^2} \right)^{(n-2)/2}$ . We have the same properties that in [B].

### **Polar Coordinates** (*Moving-Plane method*)

Now, we must use the same method than in the Theorem 1 of [B]. We will use the moving-plane method.

We must prove the lemma 2 of [B].

We set  $t \in ]-\infty, -\log 2]$  and  $\theta \in \mathbb{S}_{n-1}$  :

$$w_i(t, \theta) = e^{(n-2)t/2} u_i(y_i + e^t \theta), \quad \bar{V}_i(t, \theta) = V_i(y_i + e^t \theta) \text{ and } \bar{W}_i(t, \theta) = W_i(y_i + e^t \theta).$$

We consider the following operator  $L = \partial_{tt} - \Delta_\sigma - \frac{(n-2)^2}{4}$ , with  $\Delta_\sigma$  the Laplace-Baltrami operator on  $\mathbb{S}_{n-1}$ .

The function  $w_i$  is solution of:

$$-Lw_i = \bar{V}_i w_i^{N-1} + \epsilon_i e^{[(n+2)-(n-2)\alpha]t/2} \bar{W}_i w_i^\alpha.$$

For  $\lambda \leq 0$  we set :

$$t^\lambda = 2\lambda - t, \quad w_i^\lambda(t, \theta) = w_i(t^\lambda, \theta), \quad \bar{V}_i^\lambda(t, \theta) = \bar{V}_i(t^\lambda, \theta) \text{ et } \bar{W}_i^\lambda(t, \theta) = \bar{W}_i(t^\lambda, \theta).$$

**Remark:** Here we work on  $[\lambda, t_i] \times \mathbb{S}_{n-1}$ , with  $\lambda \leq -\frac{2}{n-2} \log u_i(y_i) + 2$  and  $t_i \leq \log \sqrt{t_i}$ , where  $t_i$  is chooses as in the proposition.

First, like in [B], we have the following lemma:

**Lemma 1:**

Let  $A_\lambda$  be the following property:

$$A_\lambda = \{\lambda \leq 0, \exists (t_\lambda, \theta_\lambda) \in ]\lambda, t_i] \times \mathbb{S}_{n-1}, \bar{w}_i^\lambda(t_\lambda, \theta_\lambda) - \bar{w}_i(t_\lambda, \theta_\lambda) \geq 0\}.$$

Then, there is  $\nu \leq 0$ , such that for  $\lambda \leq \nu$ ,  $A_\lambda$  is not true.

Like in the proof of the Theorem 1 of [B], we want to prove the following lemma:

**Lemma 2:**

For  $\lambda \leq 0$  we have :

$$w_i^\lambda - w_i \leq 0 \Rightarrow -L(w_i^\lambda - w_i) \leq 0,$$

on  $] \lambda, t_i] \times \mathbb{S}_{n-1}$ .

Like in [B], we have:

**A useful point:**

$\xi_i = \sup\{\lambda \leq \bar{\lambda}_i = 2 + \log \eta_i, w_i^\lambda - w_i < 0, \text{ on } ] \lambda, t_i] \times \mathbb{S}_{n-1}\}$ . The real  $\xi_i$  exists.

First, we have:

$$w_i(2\xi_i - t, \theta) = w_i[(\xi_i - t + \xi_i - \log \eta_i - 2) + (\log \eta_i + 2)],$$

the definition of  $w_i$  and the fact that,  $\xi_i \leq t$ , we obtain:

$$w_i(2\xi_i - t, \theta) = e^{[(n-2)(\xi_i - t + \xi_i - \log \eta_i - 2)]/2} e^{n-2} v_i[\theta e^2 e^{(\xi_i - t) + (\xi_i - \log \eta_i - 2)}] \leq 2^{(n-2)/2} e^{n-2} = \bar{c}.$$

**Proof of the Lemma 2:**

We know that:

$$-L(w_i^{\xi_i} - w_i) = [\bar{V}_i^{\xi_i} (w_i^{\xi_i})^{N-1} - \bar{V}_i w_i^{N-1}] + \epsilon_i [e^{\delta t \xi_i} \bar{W}_i^{\xi_i} (w_i^{\xi_i})^\alpha - e^{\delta t} \bar{W}_i w_i^\alpha],$$

with  $\delta = [(n+2) - (n-2)\alpha]/2$ .

We denote by  $Z_1$  and  $Z_2$  the following terms:

$$Z_1 = (\bar{V}_i^{\xi_i} - \bar{V}_i)(w_i^{\xi_i})^{N-1} + \bar{V}_i[(w_i^{\xi_i})^{N-1} - w_i^{N-1}],$$

and

$$Z_2 = \epsilon_i (\bar{W}_i^{\xi_i} - \bar{W}_i) (w_i^{\xi_i})^\alpha e^{\delta t^{\xi_i}} + \epsilon_i e^{\delta t^{\xi_i}} \bar{W}_i [(w_i^{\xi_i})^\alpha - w_i^\alpha] + \epsilon_i \bar{W}_i w_i^\alpha (e^{\delta t^{\xi_i}} - e^{\delta t}).$$

But, using the same method as in the proof of the theorem 1 of [B], we have:

$$w_i^{\xi_i} \leq w_i \text{ et } w_i^{\xi_i}(t, \theta) \leq \bar{c} \text{ pour tout } (t, \theta) \in [\xi_i, \log 2] \times \mathbb{S}_{n-1},$$

where  $\bar{c}$  is a positive constant not depending on  $i$  for  $\xi_i \leq \log \eta_i + 2$ ;

$$|\bar{V}_i^{\xi_i} - \bar{V}_i| \leq A(e^t - e^{t^{\xi_i}}) \text{ et } |\bar{W}_i^{\xi_i} - \bar{W}_i| \leq B(e^t - e^{t^{\xi_i}}),$$

Then,

$$Z_1 \leq A (w_i^{\xi_i})^{N-1} (e^t - e^{t^{\xi_i}}) \text{ et } Z_2 \leq \epsilon_i B ((w_i^{\xi_i})^\alpha (e^t - e^{t^{\xi_i}}) + \epsilon_i c (w_i^{\xi_i})^\alpha \times (e^{\delta t^{\xi_i}} - e^{\delta t})).$$

and,

$$-L(w_i^{\xi_i} - w_i) \leq (w_i^{\xi_i})^\alpha [(A w_i^{\xi_i}{}^{N-1-\alpha} + \epsilon_i B) (e^t - e^{t^{\xi_i}}) + \epsilon_i c (e^{\delta t^{\xi_i}} - e^{\delta t})].$$

But,  $w_i^{\xi_i} \leq \bar{c}$ , we obtain:

$$-L(w_i^{\xi_i} - w_i) \leq (w_i^{\xi_i})^\alpha [(A \bar{c}^{N-1-\alpha} + \epsilon_i B) (e^t - e^{t^{\xi_i}}) + \epsilon_i c (e^{\delta t^{\xi_i}} - e^{\delta t})]. \quad (1)$$

We must see the sign of:

$$\bar{Z} = [(A \bar{c}^{N-1-\alpha} + \epsilon_i B) (e^t - e^{t^{\xi_i}}) + \epsilon_i c (e^{\delta t^{\xi_i}} - e^{\delta t})].$$

$$\text{But } \alpha \in ]\frac{n}{n-2}, \frac{n+2}{n-2}[ , \delta = \frac{n+2-(n-2)\alpha}{2} \in ]0, 1[.$$

For  $t \leq t_i < 0$ , we have:

$$e^t \leq e^{(1-\delta)t_i} e^{\delta t}, \text{ for all } t \leq t_i.$$

and the fact that  $t^{\xi_i} \leq t$  ( $\xi_i \leq t$ ), by integration of the previous two members, we obtain:

$$e^t - e^{t^{\xi_i}} \leq \frac{e^{(1-\delta)t_i}}{\delta} (e^{\delta t} - e^{\delta t^{\xi_i}}), \text{ for all } t \leq t_i,$$

We can write:

$$(e^{\delta t^{\xi_i}} - e^{\delta t}) \leq \frac{\delta}{e^{(1-\delta)t_i}} (e^{t^{\xi_i}} - e^t).$$

Then,

$$-L(w_i^{\xi_i} - w_i) \leq (w_i^{\xi_i})^\alpha [-\frac{\epsilon_i \delta c}{e^{(1-\delta)t_i}} + A \bar{c}^{N-1-\alpha} + \epsilon_i B] (e^t - e^{t^{\xi_i}}).$$

The term  $\frac{\epsilon_i \delta c}{e^{(1-\delta)t_i}} - A \bar{c}^{N-1-\alpha} - \epsilon_i B$  is positive if:

$$\epsilon_i e^{-(1-\delta)t_i} \rightarrow +\infty,$$

then,

$$\epsilon_i^{(n-2)/2(1-\delta)} e^{-(n-2)/2t_i} \rightarrow +\infty.$$

If we take,  $t_i = -\frac{2}{3(n-2)} \log u_i(y_i)$ , we have:

$$\epsilon_i^{(n-2)/2(1-\delta)} [u_i(y_i)]^{1/3} \rightarrow +\infty.$$

It is given by our Hypothesis in the proposition.

But the Hopf Maximum principle, gives:

$$\min_{\theta \in \mathbb{S}_{n-1}} w_i(t_i, \theta) \leq \max_{\theta \in \mathbb{S}_{n-1}} w_i(2\xi_i - t_i, \theta),$$

then,

$$e^{(n-2)t_i} u_i(y_i) \min_{B_2(0)} u_i \leq c,$$

and,

$$[u_i(y_i)]^{1/3} \min_{B_2(0)} u_i \leq c,$$

Contradiction.

### **Proof of the Theorem 2.**

The proof is similar than the proof of the theorem 1. Only the end of the proof is different.

**Step 1:** The blow-up analysis give:

There is a sequence of points  $(y_i)_i$ ,  $y_i \rightarrow 0$  and two sequences of positive real numbers  $(l_i)_i$ ,  $(L_i)_i$ ,  $l_i \rightarrow 0$ ,  $L_i \rightarrow +\infty$ , such that if we set  $v_i(y) = \frac{u_i(y + y_i)}{u_i(y_i)}$ , we have:

$$0 < v_i(y) \leq \beta_i \leq 2^{(n-2)/2}, \quad \beta_i \rightarrow 1.$$

$$v_i(y) \rightarrow \left( \frac{1}{1 + |y|^2} \right)^{(n-2)/2}, \quad \text{uniformly on all compact set of } \mathbb{R}^n.$$

$$l_i^{(n-2)/2} u_i(y_i) \times \inf_{B_1} u_i \rightarrow +\infty,$$

**Step 2:** Application of the Hopf maximum principle.

We have the same notation that in the proof of the theorem 1. First, we take  $t_i = \sqrt{l_i}$  as in the Step 1 and we look to the end of the proof of the theorem 1. We replace  $A$  by  $k\epsilon_i$ . We want to proof that:

$$w_i^\lambda - w_i \leq 0 \Rightarrow -L(w_i^\lambda - w_i) \leq 0,$$

on  $[\xi_i, t_i] \times \mathbb{S}_{n-1}$ . We have the same defintion for  $\xi_i$  ( as in the proof of the theorem 1).

For  $t \leq t_i < 0$ , we have:

$$e^t \leq e^{(1-\delta)t_i} e^{\delta t}, \quad \text{for all } t \leq t_i.$$

and the fact that  $t^{\xi_i} \leq t$  ( $\xi_i \leq t$ ), by integration of the previous two members, we obtain:

$$e^t - e^{t^{\xi_i}} \leq \frac{e^{(1-\delta)t_i}}{\delta} (e^{\delta t} - e^{\delta t^{\xi_i}}), \quad \text{for all } t \leq t_i,$$

We can write:

$$(e^{\delta t^{\xi_i}} - e^{\delta t}) \leq \frac{\delta}{e^{(1-\delta)t_i}} (e^{t^{\xi_i}} - e^t).$$

Then,

$$-L(w_i^{\xi_i} - w_i) \leq (w_i^{\xi_i})^\alpha \left[ -\frac{\epsilon_i \delta c}{e^{(1-\delta)t_i}} + k\epsilon_i \bar{c}^{N-1-\alpha} + \epsilon_i B \right] (e^t - e^{t^{\xi_i}}).$$

The term  $\frac{\delta c}{e^{(1-\delta)t_i}} - k \bar{c}^{N-1-\alpha} - B$  is positive because  $t_i \rightarrow -\infty$  and  $\delta \in ]0, 1[$ .

But the Hopf Maximum principle, gives:

$$\min_{\theta \in \mathbb{S}_{n-1}} w_i(t_i, \theta) \leq \max_{\theta \in \mathbb{S}_{n-1}} w_i(2\xi_i - t_i, \theta),$$

then,

$$e^{(n-2)t_i} u_i(y_i) \min_{B_2(0)} u_i \leq c,$$

and,

$$l_i^{(n-2)/2} u_i(y_i) \min_{B_2(0)} u_i \leq c,$$

Contradiction with the step 1.

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